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Overview of Finite Elements Analysis

1.1 Introduction

Application areas are converging more and more as the use of CAx systems becomes more widespread. Thus, both the shaping (industrial design) and the actual product are increasingly being developed in parallel. Subsequent usage behavior, maintainability and recyclability of the product are the focus of today's growing discussion about the sustainability of products and are increasingly being mapped in simulations. Mechatronics combines mechanical effects and objects with electrical, electronic and information technology effects and objects, with the variety and number of mechatronic products increasing rapidly, as the growth of products in the entertainment and communications industries impressively shows.

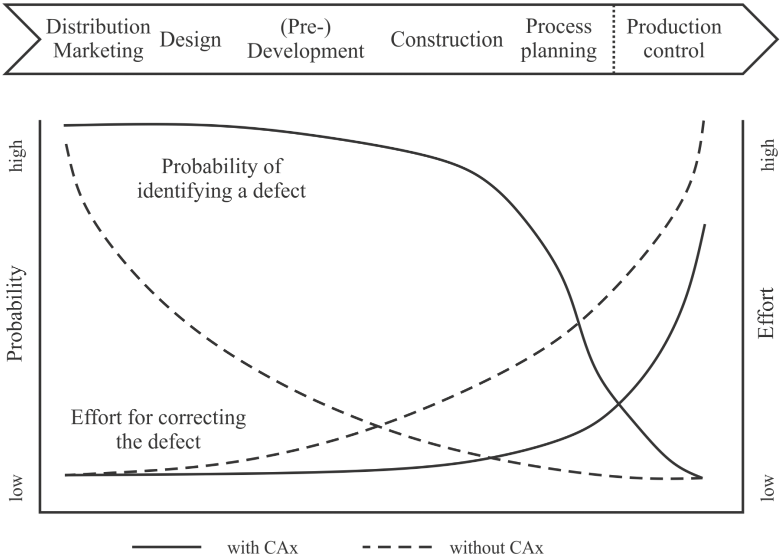

Figure 1.1 Error detection and troubleshooting (author's own figure, based on [VWZ+18])

The goal when using CAx technologies is always to make relevant decisions at the latest possible point in time during the creation of a product and to consider as many influencing factors as possible. Findings from areas other than product development are incorporated into the decision-making process. Various alternatives are simulated and evaluated as realistically as possible (see Figure 1.1). The use of CAx technologies makes it possible to identify many more potential defects at an earlier stage in the product development process. Errors can be eliminated with less effort than when CAx technologies are not used. Figure 1.1 illustrates this effect [VWZ+18].

1.2 Applications of Finite Elements Analysis

In principle, any physical or chemical problem can be considered and solved by a finite elements analysis (FEA), which can be described in terms of time- and location-dependent differential equations or an equivalent variational principle. Several well-known applications are summarized below. Most simulations using FEA focus on strength problems, potential analyses and multiphysics problems [Kle07]:

- Linear elastostatics: Hooke's material behavior ($\sigma = E \bullet \varepsilon$)
- Nonlinear elastostatics: Nonlinear material behavior (plasticity)
  - Geometric nonlinear problems (instability problems, large displacements at small strains)
  - Impulsive large deformations (crash)
  - Forming processes
- Linear elastodynamics: Natural vibrations
  - Free and forced vibrations
  - Random vibrations
- Nonlinear elastodynamics: Time- and displacement-dependent forces
  - Stability, gyroscopic motion
- Rigid body dynamics: Multibody systems (MBS)
  - Elastic multibody systems (EMBS)
- Elastohydrodynamics: Lubrication film
- Fatigue strength: Damage, fatigue life, crack fracture
- Aeroelasticity: Elastic structural behavior under incident flow
- Heat transfer: Steady-state and transient heat conduction

- Thermoelasticity: Mechanical stress under high temperatures
- Fluid flows: Seepage flow, velocity, pressure and temperature fields
- Electrical engineering: Electric flow field, magnetic and wave fields
- Acoustics: Sound pressure distribution, pressure surges
- Casting and molding: Injection molding, pressure casting, gravity casting
- Multiphysics: Coupled flow, temperature with elasticity

In the field of elastostatics and elastodynamics, either the differential equation of equilibrium or, alternatively, the equality of the internal and external virtual work are solved and calculated to serve as the calculation basis for the simulation. The analytical solution of both equations is not possible if the geometry is complex. Nevertheless, an approximate solution for the differential equations is possible if suitable approaches are adopted. However, the results are then no longer exact. They are approximations of the exact result.

The displacement magnitude method is usually applied to elastostatic and elastodynamic problems. The forces acting on a structure are known. What is unknown are the resulting displacements and deformations of the structure. The displacement behavior of the elements is given and the resulting system of equations is solved numerically.

Also possible, but uncommon, is the application of the force magnitude method. Here, the forces acting on a structure are unknown. As it is often easier to determine and describe the acting forces than the displacements, the displacement magnitude method has become widely accepted in practice.

Finite elements analysis is being developed further and further. In parallel, computer systems are becoming more powerful, which means that more complex system modeling and handling of field problems or multiphysics problems are becoming more and more important and increasingly broader fields of application are being opened up. For example, dynamic and elastodynamic systems are increasingly represented by multibody systems and elastic multibody systems. Field problems are mainly problems of heat conduction, potential flow and magnetism. These can be described by an identical type of differential equation. Heat conduction problems are represented by the Fourier heat conduction equation. For potential flows, Poisson's equation for potential flows is used. Magnetic force effects can be simulated using Maxwell's equation. Computational fluid dynamics (CFD) programs can model flow problems in air or water or viscous media, such as plastics [Kle07].

1.3 Basics of Model Building

An essential principle of systems thinking is to represent systems and complex interrelationships by means of models. Models are simplifications and abstractions of reality. Therefore, they show only the necessary partial aspects. Thus, it is important that the models be sufficiently meaningful with respect to the situation and the problem. In all considerations, the question of usefulness and problem relevance must be asked [DH02].

Model building is used specifically to solve problems. In this process, the problem-solving process is shifted from the initial level (e.g., reality) to an abstract level, so that finding a solution is usually easier. On the abstract model level, solutions are sought and worked out with the help of abstracted models. The goal is for the solution or interpretation of the model to have as high a relevance (validity) as possible for the solution of the original (original, e.g. actual) problem. Thus, model building is an important problem-solving technique in the sense of goal-oriented simplification through reducing an (not necessarily actual) original to the necessary by abstraction (see Figure 1.2) [VWZ+18].

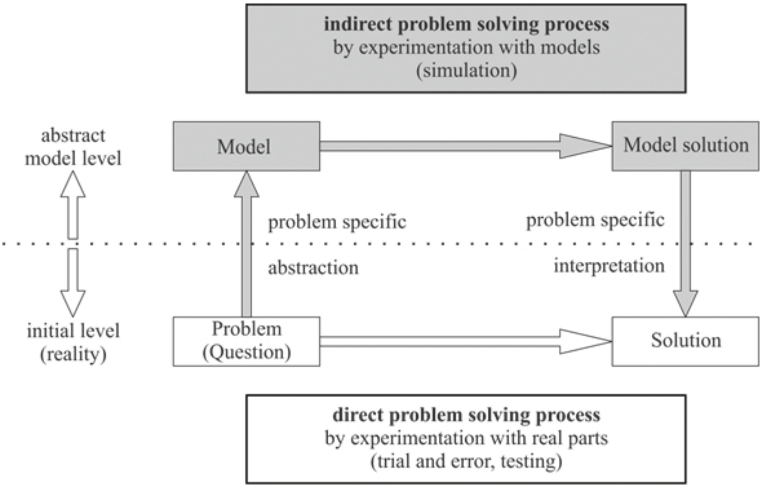

Figure 1.2 Direct and indirect problem-solving through problem-specific abstraction (author's own figure, based on [VWZ+18])

In the technical and scientific environment, a concept of reality is necessary which allows of the possibility that technical entities exist in reality and that at least a part of the truth about their reality can be represented by measurement results. Otherwise, the observation of regularities and the making of predictions about these entities are not possible. Science translates the reality observed by perceptions, hypotheses and modeling into symbols of a theory language, such as mathematical formalization.

In connection with theories from the hypotheses and models, scientific data about this reality arise. However, the data acquire their meaning or interpretation only in connection with the respective hypotheses and models [DH02].

1.3.1 Requirements Imposed on Models

Models must closely resemble the initial object or situation. They must correspond to characteristic properties that describe the purpose of the investigation. Given errors are usually tolerable or are consciously accepted. In product development, the purpose of the model is heavily dependent on the respective life phase of the product on which the investigation is based. A suitable cost/benefit ratio is desirable. The level of detail of the model is closely related to the effort expended on the modeling and the subsequent analysis. In many cases, a very "detailed" model is not necessary, since the uncertainties of a detailed model can be so large that its usefulness must be questioned when compared with a simpler model. A model must be clearly defined, unambiguously describable, internally consistent, free of redundancy, and manageable in order to be easily used to solve a particular task [Rod06].

In a specified domain of validity, the behavior of a model must correspond to the behavior of the actual system (model validity). This behavior is the result of the characteristic properties of the model elements as well as their interconnections. If different possibilities for modeling a system exist which satisfy the requirements mentioned above, then the simplest possibility should be given preference (model efficiency). For the creation of a simple, efficient and valid model there are no generally applicable rules. Experience and the prior knowledge of the model builder therefore play a large role [VWZ+18].

1.3.2 Methods of Model Building

In the practical application of models, various methods of model building have become established and accepted [Ise99, Rod06, HGP07].

Computational Methods

For computational methods, mathematical models are required which are described by algebraic equations, differential equations and the like. For the solution of the mathematical models, powerful numerical and symbolic software programs are available today, in addition to the traditional analytical methods. The advantage of the computational methods is that neither real structures nor physical models are necessary. Model variants, for example, due to design changes, can be investigated with little effort. The execution of parameter studies and optimizations is comparatively simple. The idealizations and simplifications necessary for model building have a strong effect on the quality of the results.

Today, computational methods are very well developed. Nevertheless, physical experiments cannot be completely dispensed with, as the validity of the mathematical models needs to be ensured. The optimization of products, parameter studies and the establishment of general relationships (approximate solutions) are typical areas of application. CAx systems such as FEA systems, MBS simulation tools, CAD systems or computer algebra tools are used for the application.

Experimental or Measurement Methods

Experimental or measurement methods require physical, solid models for experiments. Tests, measurements and evaluations can be carried out on these. Typically, they are prototypes, test objects, test setups or scale models, which are often used in mechanical engineering. The use of physical models makes it possible to measure all significant influences.

Measurements are performed on the real structure. The recorded signals are only influenced by possible measurement errors. Only directly measurable parameters can be recorded. Internal state variables or those inaccessible to the measuring technique cannot be recorded when only this method is used. They remain hidden and this creates the problem that the overall system can only be partially recorded and described. Parameter studies are carried out to capture interrelationships. These require a great deal of effort. Experimental or measurement methods are mainly used for engine test-runs, prototype analysis, vibration monitoring, early damage detection and diagnosis, and the verification of computational results (sampling).

Hybrid Methods (Combinations of Computation and Experiment or Measurement)

These methods use both computational models and measured parameters. With computational methods, errors occur mainly due to modeling inaccuracies. With experimental methods, more or less large measurement errors cannot be ruled out. If results from the mathematical procedures and the experimental or measurement procedures are available, hypotheses can be made about the nature of the errors. The model can then be improved in such a way that better agreement between calculation and experiment is obtained. The resulting hybrid procedures are assigned to the identification procedures and are separated into parameter identification and model identification. In parameter identification, only the parameters of an existing mathematical model are reconstructed from measurement data, while in model identification the measurement data (also) serve to set up a mathematical model itself – including its structure. These methods are widely used in control engineering and in quality control [VWZ+18].

The process for building a computer internal model for model analysis is summarized in Figure 1.3. First, the model builder develops a mental model of the original to be analyzed. This can be a real technical object or a new product. The result is the mental model (thought model). This is formalized by information elements and structures that allow it to be captured in the form of data. This "information model" is implemented on the computer (computational model, see [PBF+07, DA95]). Central relevance thereby accrues to the mental model. It already contains the necessary abstractions and represents the starting point for efficient formalization. The latter is crucial for successful implementation of the model on a computer. Usually, models are optimized in several iteration loops. Depending on the model accuracy, several iteration loops may be necessary. The further development of software, hardware and methods offers more and more possibilities. This provides important feedback on the entire model building process, which can even influence and change the original itself. This trend is impressively confirmed by the developments around cyber-physical systems or Industry 4.0 [VWZ+18].

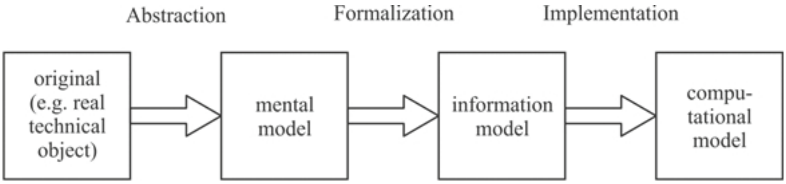

Figure 1.3 Generation of a computational model (author's own figure, based on [VWZ+18])

1.3.3 Requirements Imposed on the Model Builder

The model builder is the person who generates the model. He therefore needs knowledge and experience both of the problem to be investigated and of model building/simulation or test and measurement methods. His working method should be systematic and method-supported in order that he may successfully meet the requirements specification. Good modeling always means "leaving out the right stuff."

The model builder should have the following basic qualifications:

- In-depth knowledge of the system under investigation. The model builder must decide what can be neglected and is therefore not considered further in the investigations.

- Advanced knowledge and expertise of the tools and methods available for modeling and simulation or experimental and measurement methods.
- Experience in the selection of suitable models, taking into account the costs, time and informative value of the model results.
- Creativity in the generation, limitation, and definition of the model.
- Practice in interpreting results: results must be interpreted “correctly”. Among other things, it must be possible to distinguish between physical effects and artifacts (measurement errors or numerical effects).

The choice of tools for modeling depends on the task, the application area and the benefit/effort ratio. Many computer-based tools are available. Simulations are classified by the type of model used. These can be physical, numerical-analytical or graphical models. A sharp distinction between the different types of simulations is barely possible in practice. Depending on the accessibility of the calculation task, different methods are used. Different methods often complement each other synergistically. Thus, measurements on real objects can be carried out to identify model parameters or numerical models can be checked by known analytical solutions. According to this system, calculation can be understood as a simulation by means of analytical or numerical models [VWZ+18].

Simulations are carried out to answer a wide variety of questions. The following list does not claim to be complete:

- No real system is available (e.g. in the design phase).
- Experiments on the real system take too long.
- Experiments on the real system are too expensive (e.g. for a crash test).
- Experiments on the real system are too dangerous (e.g. for airplanes, power plants).
- The time constants of the real system are too large (e.g. climate models).
- The test scenarios (“load cases”) cannot be controlled.

Figure 1.4 shows the relationship between simulation and modeling. In addition to the corresponding models, solution methods and representation models are required to carry out a simulation. The common simulation systems offer extensible libraries and interfaces for importing and exporting data. Input and generation of models are done graphically and functions can be visualized.

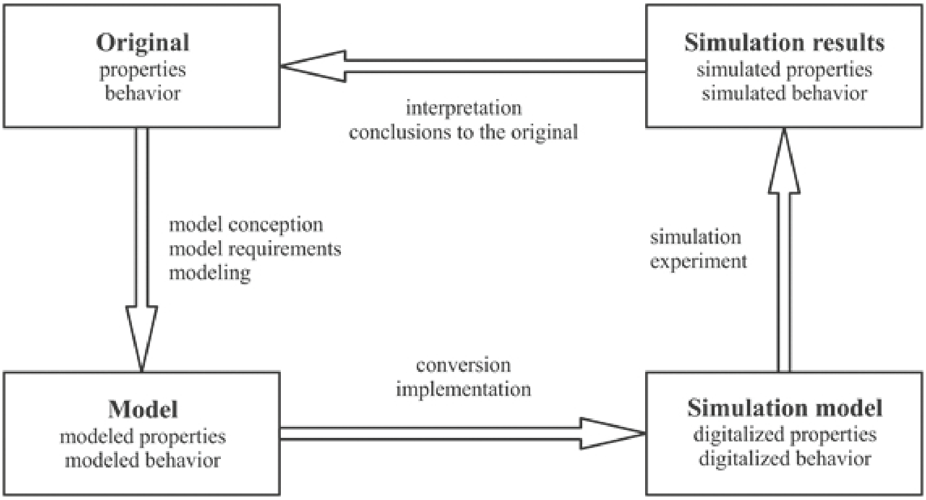

Figure 1.4 Simulation cycle with models (author's own figure, based on [VWZ+18])

1.3.4 Model Validation and Model Verification

The terms verification and validation are described below. They must not be confused [VDI99, VDI03]. The classification of verification and validation in the modeling and simulation process is shown in Figure 1.5.

Verification

Verification checks whether the specified requirements are met by the model. The implementation of the model in the simulation software is also checked. The model is verified with the help of simple calculations. Empirical or third-party experience serves as a basis for the verification. Test calculations, such as systematic experiments or consistency checks, are used to verify whether the basic model behaves plausibly. Internal consistency is checked. If inconsistencies are found, it must be investigated whether the model is faulty or whether the fault lies in the expectations about the behavior of the real system. Verification is therefore only done for the model behavior and is independent of comparisons with the original system. Often, verification takes the form of sensitivity analyses. Individual boundary conditions, such as applied loads, geometric parameters or material parameters are changed and the behavior of the model is evaluated for plausibility. If minor changes to the described parameters lead to implausible changes in the results, the model must be critically questioned.

Validation

While verification refers to the quality of the model, validation checks whether the real system is satisfactorily simulated by the model generated. The limits of the model, meaning the range in which the model is valid, are also determined during validation. Thus, it must be ensured that the model can represent the behavior of the real system accurately enough and without errors, with regard to the investigation objectives. The question is answered as to whether the right thing was done. Special attention must be paid to the first simulation runs, as these serve to validate the simulation model. Complete agreement of the simulation model with the real system to be represented is impossible. But it is also unnecessary. Like verification, validation can also be achieved with a sensitivity analysis. The behavior of the model is examined when the load cases or individual parameters of the model are changed. In contrast to verification, validation involves a comparison with the behavior of the real system. In addition to the sensitivity analysis, plausibility checks are carried out. This examines the value ranges of the input and results data and the consistency of the physical unities in terms the real systems to be examined. It also includes a comparison with measurements on the real object or on a prototype [VWZ+18].

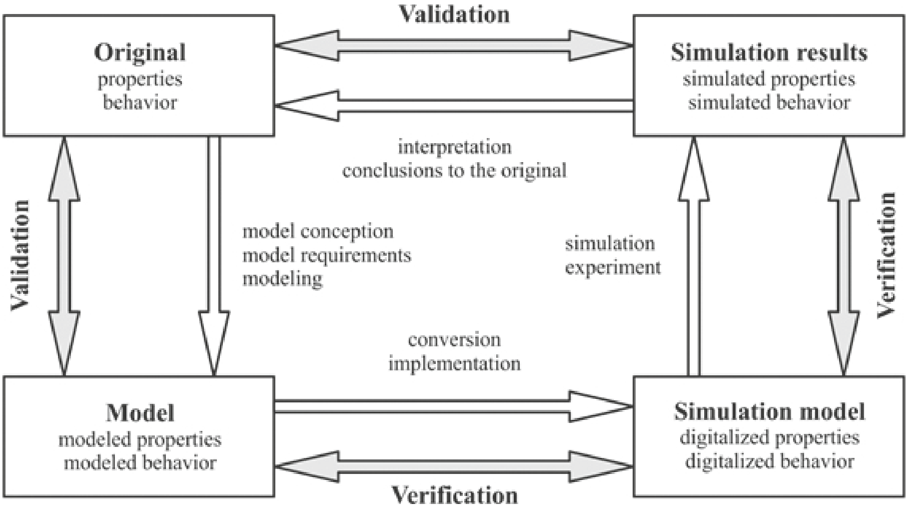

Figure 1.5 Simulation cycle with models (author's own figure, based on [VWZ+18])

1.4 Finite Elements Analysis

Finite elements analysis is one of the most important numerical computational methods and is one of the most frequently used in engineering. The first applications were in physically based mathematical models of stress and deformation problems in structural mechanics. Based on this, finite elements analysis was extended to the field of continuum mechanics.

FEA is an approximation method and is used to solve problems in engineering and physics. The approximation uses mathematical models in which solid or fluid bodies are divided into elements of finite size (“finite elements”). Suitable conditions for the transitions between the individual elements must be defined at the element boundaries. It is important that the sum of all elements in connection with the transition conditions correspond to the overall model. Dividing a body into finite elements makes it possible to approximate complex geometries with “any precision”. The extreme principle selected for the computation (e.g. minimum of the potential energy) applies both to the global model and to the individual finite elements. For the solution of the computation problem, first an adequate mathematical model is selected. This is described by algebraic equations, ordinary or partial differential equations or by a combination of them. The equations can have any form and be linear or nonlinear. Both stationary (unchanging in time, in particular static) and transient (changing in time, transient, dynamic) processes or systems can be considered as problems [VWZ+18].

With the increase in performance of computer systems in recent decades, FEA has been brought to bear in many different engineering disciplines. Applications include strength calculations, the dimensioning of machine elements and the calculation of magnetic fields. FEA simulation is a numerical experiment and offers some advantages over physical experiments [VWZ+18]:

- Time and cost savings (reduction of the time and effort associated with prototype construction for planning, execution and evaluation of tests)
- Calculation proofs are increasingly required as quality proofs
- Possibility of performing cost-effective and fast variant studies and parameter variations on the computer-internal model
- Analysis of areas that are difficult or impossible to access for measurements (e.g. engine combustion chamber, blast furnace, steam turbine, parts in casting, forming or machining processes, structural elements in crash tests)
- Analysis of systems on which tests are not possible, too dangerous or too expensive (e.g. earthquake loading of large structures)
- Determination and analysis of complete two- or three-dimensional distributions of physical quantities (stresses, displacements, support reactions, etc.)

However, FEA cannot answer all questions, and so experiments are still necessary. Calculation results must be verified on real models. The computational methods are subject to continuous improvement. At present, FEA systems are mainly used in engineering disciplines. A wide variety of problems are analyzed and answered [VWZ+18].

Today, FEA systems are used intensively in almost all industries, especially in aerospace, automotive and general mechanical engineering (machine tools, steel construction, shipbuilding, etc). FEA systems are also used in the plastics, consumer goods, electrical and electronics industries.

Despite the widespread use of FEA systems in practical applications, the correct application of these systems requires a qualified engineer, usually a specialist. In the past, a distinction was made between CAD designers and FEA engineers, although today these two fields of activity are increasingly merging. Despite all the simplifications, FEA problems are not automatically solved by computers. Table 1.1 summarizes an activity analysis, showing that the computer is only the central tool, without whose performance the method would generally not be economically viable.

Table 1.1 Activity analysis for processing FE problems (source: [Kle07])

Processing steps required	Estimated time required	Estimated computing time
Methodical preparation of the problem	10%	
Generation of an FE model in the pre-processor	50%	20%
Computational run		70%
Results evaluation in the post-processor and documentation	30%	10%
Plausibility check	10%	

■ 1.5 Basic Rules for the Correct Use of FEA

To complete the introduction of finite elements analysis, some basic principles of its application will be discussed. These should be considered when preparing simulation studies, since their non-observance leads either to errors or to unnecessary rework. In practice, the applications and thus the problems that arise are naturally very complex, and so the most important sources of error are considered below [Kle07].

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